Singapore Mathematical Society

Singapore Mathematical Olympiad (SMO) 2010

(Junior Section, Round 2)

Saturday, 25 June 2010 **0930-1230**

INSTRUCTIONS TO CONTESTANTS

1. *Answer ALL 5 questions.*
2. *Show all the steps in your working.*
3. *Each question carries 10 mark.*
4. *No calculators are allowed.*

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1. Let the diagonals of the square $ABCD$ intersect at S and let P be the midpoint of AB. Let M be the intersection of AC and PD and N the intersection of BD and PC. A circle is incribed in the quadrilateral $PMSN$. Prove that the radius of the circle is $MP - MS$.

2. Find the sum of all the 5-digit integers which are not multiples of 11 and whose digits are 1, 3, 4, 7, 9.

3. Let $a_1, a_2, \ldots, a_n$ be positive integers, not necessarily distinct but with at least five distinct values. Suppose that for any $1 \leq i < j \leq n$, there exist k, ℓ, both different from i and j such that $a_i + a_j = a_k + a_\ell$. What is the smallest possible value of n?

4. A student divides an integer m by a positive integer n, where $n \leq 100$, and claims that

$$\frac{m}{n} = 0 \cdot 167 a_1 a_2 \cdots.$$

Show the student must be wrong.

5. The numbers $\frac{1}{1}, \frac{1}{2}, \ldots, \frac{1}{2010}$ are written on a blackboard. A student chooses any two of the numbers, say x, y, erases them and then writes down $x + y + xy$. He continues to do this until only one number is left on the blackboard. What is this number?